

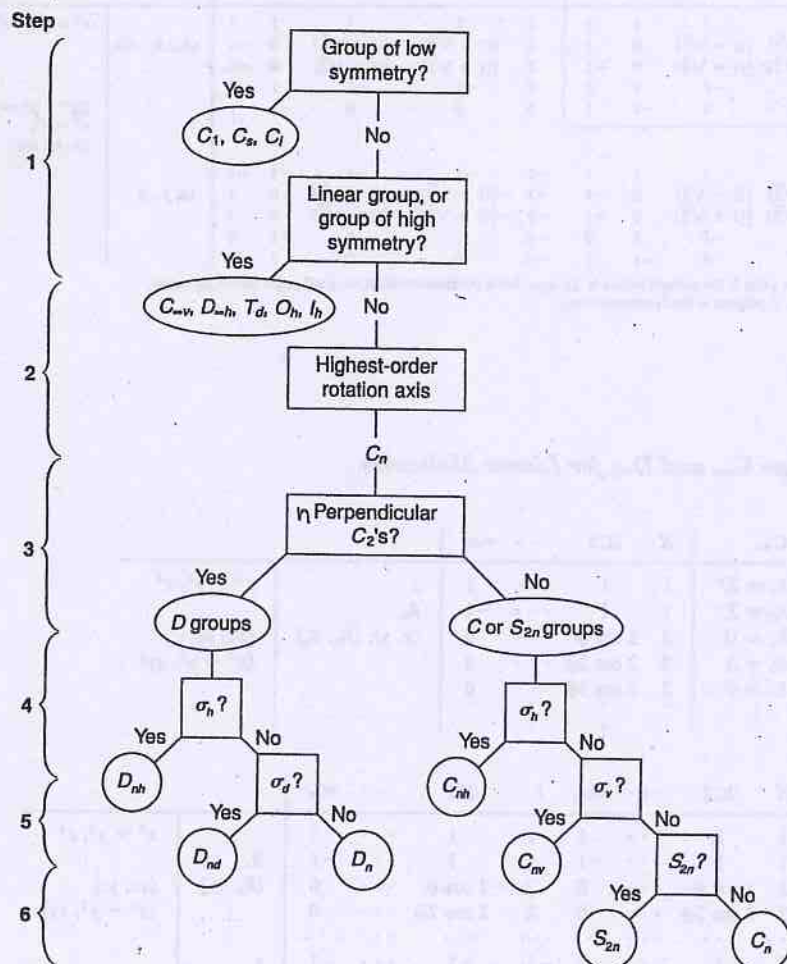


Figure 9.7
Diagram of procedure for assigning point groups. [Adapted from G. L. Miessler and D. A. Tarr, *Inorganic Chemistry*, Prentice Hall, Englewood Cliffs, NJ, 1991; p. 97.]

APPENDIX C

Character Tables

THE NONAXIAL GROUPS

C_1	E		
A	1		
C_2	E	σ_h	
A'	1	1	x, y, R_z
A''	1	-1	z, R_x, R_y
			$x^2, y^2, z^2, xy, yz, xz$
C_i	E	i	
A_g	1	1	R_x, R_y, R_z
A_u	1	-1	x, y, z
			$x^2, y^2, z^2, xy, xz, yz$

THE AXIAL GROUPS

► The C_n Groups

C_2	E	C_2		
A	1	1	z, R_z	x^2, y^2, z^2, xy
B	1	-1	x, y, R_x, R_y	yz, xz
C_3	E	C_3	C_3^2	$\varepsilon = \exp(2\pi i/3)$
A	1	1	1	z, R_z
E	$\begin{Bmatrix} 1 & \varepsilon & \varepsilon^* \\ 1 & \varepsilon^* & \varepsilon \end{Bmatrix}$			$(x, y), (R_x, R_y)$
				$x^2 + y^2, z^2$
				$(x^2 - y^2, xy), (yz, xz)$

C_4	E	C_4	C_2	C_4^3		
A	1	1	1	1	z, R_z	$x^2 + y^2, z^2$
B	1	-1	1	-1		$x^2 - y^2, xy$
E	$\begin{Bmatrix} 1 & i \\ 1 & -i \end{Bmatrix}$	$\begin{Bmatrix} i & -1 \\ -i & 1 \end{Bmatrix}$			$(x, y), (R_x, R_y)$	(xz, yz)

C_5	E	C_5	C_5^2	C_5^3	C_5^4	$\varepsilon = \exp(2\pi i/5)$	
A	1	1	1	1	1	z, R_z	$x^2 + y^2, z^2$
E_1	$\begin{Bmatrix} 1 & \varepsilon & \varepsilon^2 & \varepsilon^{2*} & \varepsilon^* \\ 1 & \varepsilon^* & \varepsilon^{2*} & \varepsilon^2 & \varepsilon \end{Bmatrix}$					$(x, y), (R_x, R_y)$	(yz, xz)
E_2	$\begin{Bmatrix} 1 & \varepsilon^2 & \varepsilon^* & \varepsilon & \varepsilon^{2*} \\ 1 & \varepsilon^{2*} & \varepsilon & \varepsilon^* & \varepsilon^2 \end{Bmatrix}$						$(x^2 - y^2, xy)$

C_6	E	C_6	C_3	C_2	C_3^2	C_6^5	$\varepsilon = \exp(2\pi i/6)$	
A	1	1	1	1	1	1	z, R_z	$x^2 + y^2, z^2$
B	1	-1	1	-1	1	-1		
E_1	$\begin{Bmatrix} 1 & \varepsilon & -\varepsilon^* & -1 & -\varepsilon & \varepsilon^* \\ 1 & \varepsilon^* & -\varepsilon & -1 & -\varepsilon^* & \varepsilon \end{Bmatrix}$						$(x, y),$ (R_x, R_y)	(xz, yz)
E_2	$\begin{Bmatrix} 1 & -\varepsilon^* & -\varepsilon & 1 & -\varepsilon^* & -\varepsilon \\ 1 & -\varepsilon & -\varepsilon^* & 1 & -\varepsilon & -\varepsilon^* \end{Bmatrix}$							$(x^2 - y^2, xy)$

C_7	E	C_7	C_7^2	C_7^3	C_7^4	C_7^5	C_7^6	$\varepsilon = \exp(2\pi i/7)$
A	1	1	1	1	1	1	1	z, R_z
E_1	$\begin{Bmatrix} 1 & \varepsilon & \varepsilon^2 & \varepsilon^3 & \varepsilon^{3*} & \varepsilon^{2*} & \varepsilon^* \\ 1 & \varepsilon^* & \varepsilon^{2*} & \varepsilon^{3*} & \varepsilon^3 & \varepsilon^2 & \varepsilon \end{Bmatrix}$							$x^2 + y^2, z^2$ $(x, y), (R_x, R_y)$ (xz, yz)
E_2	$\begin{Bmatrix} 1 & \varepsilon^2 & \varepsilon^{3*} & \varepsilon^* & \varepsilon & \varepsilon^3 & \varepsilon^{2*} \\ 1 & \varepsilon^{2*} & \varepsilon^3 & \varepsilon^* & \varepsilon & \varepsilon^{3*} & \varepsilon^2 \end{Bmatrix}$							$(x^2 - y^2, xy)$
E_3	$\begin{Bmatrix} 1 & \varepsilon^3 & \varepsilon^* & \varepsilon^2 & \varepsilon^{2*} & \varepsilon & \varepsilon^{3*} \\ 1 & \varepsilon^{3*} & \varepsilon & \varepsilon^{2*} & \varepsilon^2 & \varepsilon^* & \varepsilon^3 \end{Bmatrix}$							

C_3	E	C_6	C_4	C_2	C_3^2	C_6^5	C_4^3	C_2^3	$\varepsilon = \exp(2\pi i/8)$	
A	1	1	1	1	1	1	1	1	z, R_z	$x^2 + y^2, z^2$
B	1	-1	1	1	1	-1	-1	-1		
E_1	$\begin{Bmatrix} 1 & \varepsilon & i & -1 & -i & -\varepsilon^* & -\varepsilon & \varepsilon^* \\ 1 & \varepsilon^* & -i & -1 & i & -\varepsilon & -\varepsilon^* & \varepsilon \end{Bmatrix}$	$(x, y),$ (R_x, R_y)	(xz, yz)							
E_2	$\begin{Bmatrix} 1 & i & -1 & 1 & -1 & -i & i & -i \\ 1 & -i & -1 & 1 & -1 & i & -i & i \end{Bmatrix}$		$(x^2 - y^2, xy)$							
E_3	$\begin{Bmatrix} 1 & -\varepsilon & i & -1 & -i & \varepsilon^* & \varepsilon & -\varepsilon^* \\ 1 & -\varepsilon^* & -i & -1 & i & \varepsilon & \varepsilon^* & -\varepsilon \end{Bmatrix}$									

► The Icosahedral Groups^a

I_1	E	$12C_2$	$12C_3$	$20C_3$	$15C_2$	I	$12S_{10}$	$12S_{10}^*$	$20S_6$	15σ		
A_g	1	1	1	1	1	1	1	1	1	1	(R_x, R_y, R_z)	$x^2 + y^2 + z^2$
T_{1g}	3	$\frac{1}{2}(1 + \sqrt{5})$	$\frac{1}{2}(1 - \sqrt{5})$	0	-1	3	$\frac{1}{2}(1 - \sqrt{5})$	$\frac{1}{2}(1 + \sqrt{5})$	0	-1		
T_{2g}	3	$\frac{1}{2}(1 - \sqrt{5})$	$\frac{1}{2}(1 + \sqrt{5})$	0	-1	3	$\frac{1}{2}(1 + \sqrt{5})$	$\frac{1}{2}(1 - \sqrt{5})$	0	-1		
G_g	4	-1	-1	1	0	4	-1	-1	1	0		
H_g	5	0	0	-1	1	5	0	0	-1	1		
A_u	1	1	1	1	1	-1	-1	-1	-1	-1	(x, y, z)	$(2z^2 - x^2 - y^2, x^2 - y^2, xy, yz, zx)$
T_{1u}	3	$\frac{1}{2}(1 + \sqrt{5})$	$\frac{1}{2}(1 - \sqrt{5})$	0	-1	-3	$-\frac{1}{2}(1 - \sqrt{5})$	$-\frac{1}{2}(1 + \sqrt{5})$	0	1		
T_{2u}	3	$\frac{1}{2}(1 - \sqrt{5})$	$\frac{1}{2}(1 + \sqrt{5})$	0	-1	-3	$-\frac{1}{2}(1 + \sqrt{5})$	$-\frac{1}{2}(1 - \sqrt{5})$	0	1		
G_u	4	-1	-1	1	0	-4	1	1	-1	0		
H_u	5	0	0	-1	1	-5	0	0	1	-1		

* For the pure rotation group I, the outlined section in the upper left is the character table; the g subscripts should, of course, be dropped and (x, y, z) assigned to the T_1 representation.

► The Groups $C_{\infty v}$ and $D_{\infty h}$ for Linear Molecules

C_m	E	$2C_2^*$	$\dots$	∞C_σ		
$A_1 = \Sigma^+$	1	1	$\dots$	1	z	$x^2 + y^2, z^2$
$A_2 = \Sigma^-$	1	1	$\dots$	-1	R_z	
$E_1 = \Pi$	2	$2 \cos \phi$	$\dots$	0	$(x, y); (R_x, R_y)$	(xz, yz)
$E_2 = \Delta$	2	$2 \cos 2\phi$	$\dots$	0		$(x^2 - y^2, xy)$
$E_3 = \Phi$	2	$2 \cos 3\phi$	$\dots$	0		
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$		

[illegible]

T_h	E	$4C_3$	$4C_3^2$	$3C_2$	i	$4S_6$	$4S_6^5$	$3\sigma_h$	$(\varepsilon = \exp(2\pi i/3))$
A_g	1	1	1	1	1	1	1	1	$x^2 + y^2 + z^2$
A_u	1	1	1	1	-1	-1	-1	-1	
E_g	$\begin{Bmatrix} 1 & \varepsilon & \varepsilon^* \\ 1 & \varepsilon^* & \varepsilon \end{Bmatrix}$	$\begin{Bmatrix} \varepsilon^* & 1 \\ \varepsilon & \varepsilon^* \end{Bmatrix}$	$\begin{Bmatrix} 1 & 1 \\ \varepsilon & \varepsilon^* \end{Bmatrix}$	$\begin{Bmatrix} 1 & 1 \\ -1 & -1 \end{Bmatrix}$	$\begin{Bmatrix} \varepsilon & \varepsilon^* \\ -\varepsilon & -\varepsilon^* \end{Bmatrix}$	$\begin{Bmatrix} \varepsilon^* & 1 \\ \varepsilon & \varepsilon^* \end{Bmatrix}$	$\begin{Bmatrix} 1 & 1 \\ \varepsilon & \varepsilon^* \end{Bmatrix}$	$\begin{Bmatrix} 1 & 1 \\ \varepsilon & \varepsilon^* \end{Bmatrix}$	$\begin{Bmatrix} 2z^2 - x^2 - y^2 \\ x^2 - y^2 \end{Bmatrix}$
E_u	$\begin{Bmatrix} 1 & \varepsilon & \varepsilon^* \\ 1 & \varepsilon^* & \varepsilon \end{Bmatrix}$	$\begin{Bmatrix} \varepsilon^* & 1 \\ \varepsilon & \varepsilon^* \end{Bmatrix}$	$\begin{Bmatrix} 1 & 1 \\ \varepsilon & \varepsilon^* \end{Bmatrix}$	$\begin{Bmatrix} 1 & 1 \\ -1 & -1 \end{Bmatrix}$	$\begin{Bmatrix} \varepsilon & \varepsilon^* \\ -\varepsilon & -\varepsilon^* \end{Bmatrix}$	$\begin{Bmatrix} \varepsilon^* & 1 \\ \varepsilon & \varepsilon^* \end{Bmatrix}$	$\begin{Bmatrix} 1 & 1 \\ \varepsilon & \varepsilon^* \end{Bmatrix}$	$\begin{Bmatrix} 1 & 1 \\ \varepsilon & \varepsilon^* \end{Bmatrix}$	$\begin{Bmatrix} 2z^2 - x^2 - y^2 \\ x^2 - y^2 \end{Bmatrix}$
T_g	3	0	0	-1	3	0	0	-1	(R_x, R_y, R_z)
T_u	3	0	0	-1	-3	0	0	1	(x, y, z)

T_d	E	$8C_3$	$3C_2$	$6S_4$	$6\sigma_d$	
A_1	1	1	1	1	1	$x^2 + y^2 + z^2$
A_2	1	1	1	-1	-1	
E	2	-1	2	0	0	$(2z^2 - x^2 - y^2, x^2 - y^2)$
T_1	3	0	-1	1	-1	(R_x, R_y, R_z)
T_2	3	0	-1	-1	1	(x, y, z)

► Octahedral Groups

O	E	$6C_4$	$3C_2(=C_4^2)$	$8C_3$	$6C_2$	
A_1	1	1	1	1	1	$x^2 + y^2 + z^2$
A_2	1	-1	1	1	-1	
E	2	0	2	-1	0	$(2z^2 - x^2 - y^2, x^2 - y^2)$
T_1	3	1	-1	0	-1	$(R_x, R_y, R_z), (x, y, z)$
T_2	3	-1	-1	0	1	(xy, xz, yz)

O_h	E	$8C_3$	$6C_2$	$6C_4$	$3C_2(=C_4^2)$	i	$6S_4$	$8S_6$	$3\sigma_h$	$6\sigma_d$	
A_{1g}	1	1	1	1	1	1	1	1	1	1	$x^2 + y^2 + z^2$
A_{2g}	1	1	-1	-1	1	1	-1	1	1	-1	
E_g	2	-1	0	0	2	2	0	-1	2	0	$(2z^2 - x^2 - y^2, x^2 - y^2)$
T_{1g}	3	0	-1	1	-1	3	1	0	-1	-1	(R_x, R_y, R_z)
T_{2g}	3	0	1	-1	-1	3	-1	0	-1	1	(xz, yz, xy)
A_{1u}	1	1	1	1	1	-1	-1	-1	-1	-1	
A_{2u}	1	1	-1	-1	1	-1	1	-1	-1	1	
E_u	2	-1	0	0	2	-2	0	1	-2	0	
T_{1u}	3	0	-1	1	-1	-3	-1	0	1	1	(x, y, z)
T_{2u}	3	0	1	-1	-1	-3	1	0	1	-1	

Character Tables

► The S_n Groups

S_4	E	S_4	C_2	S_4^3	
A	1	1	1	1	R_i
B	1	-1	1	-1	z
E	$\begin{Bmatrix} 1 & i & -1 & -i \\ 1 & -i & -1 & i \end{Bmatrix}$	$\begin{Bmatrix} \varepsilon & \varepsilon^* & \varepsilon & \varepsilon^* \\ \varepsilon^* & \varepsilon & \varepsilon^* & \varepsilon \end{Bmatrix}$	$\begin{Bmatrix} 1 & 1 \\ \varepsilon & \varepsilon^* \end{Bmatrix}$	$\begin{Bmatrix} 1 & 1 \\ \varepsilon & \varepsilon^* \end{Bmatrix}$	$\begin{Bmatrix} x^2 + y^2, z^2 \\ x^2 - y^2, xy \end{Bmatrix}$

S_6	E	C_3	C_3^2	i	S_6^5	S_6	$\varepsilon = \exp(2\pi i/3)$
A_1	1	1	1	1	1	1	R_i
E_g	$\begin{Bmatrix} 1 & \varepsilon & \varepsilon^* \\ 1 & \varepsilon^* & \varepsilon \end{Bmatrix}$	$\begin{Bmatrix} \varepsilon^* & 1 \\ \varepsilon & \varepsilon^* \end{Bmatrix}$	$\begin{Bmatrix} 1 & 1 \\ \varepsilon & \varepsilon^* \end{Bmatrix}$	$\begin{Bmatrix} 1 & 1 \\ \varepsilon & \varepsilon^* \end{Bmatrix}$	$\begin{Bmatrix} 1 & 1 \\ \varepsilon & \varepsilon^* \end{Bmatrix}$	$\begin{Bmatrix} 1 & 1 \\ \varepsilon & \varepsilon^* \end{Bmatrix}$	$\begin{Bmatrix} x^2 + y^2, z^2 \\ (x^2 - y^2, xy), (xy, yz) \end{Bmatrix}$
A_u	1	1	1	-1	-1	-1	z
E_u	$\begin{Bmatrix} 1 & \varepsilon & \varepsilon^* \\ 1 & \varepsilon^* & \varepsilon \end{Bmatrix}$	$\begin{Bmatrix} \varepsilon^* & 1 \\ \varepsilon & \varepsilon^* \end{Bmatrix}$	$\begin{Bmatrix} 1 & 1 \\ \varepsilon & \varepsilon^* \end{Bmatrix}$	$\begin{Bmatrix} 1 & 1 \\ \varepsilon & \varepsilon^* \end{Bmatrix}$	$\begin{Bmatrix} 1 & 1 \\ \varepsilon & \varepsilon^* \end{Bmatrix}$	$\begin{Bmatrix} 1 & 1 \\ \varepsilon & \varepsilon^* \end{Bmatrix}$	(x, y)

S_8	E	S_8	C_4	S_8^3	C_2	S_8^5	C_4^3	S_8^7	$\varepsilon = \exp(2\pi i/8)$
A	1	1	1	1	1	1	1	1	R_i
B	1	-1	1	-1	1	-1	1	-1	z
E_1	$\begin{Bmatrix} 1 & \varepsilon & i & -\varepsilon^* \\ 1 & \varepsilon^* & -i & -\varepsilon \end{Bmatrix}$	$\begin{Bmatrix} \varepsilon^* & 1 \\ \varepsilon & \varepsilon^* \end{Bmatrix}$	$\begin{Bmatrix} 1 & 1 \\ \varepsilon & \varepsilon^* \end{Bmatrix}$	$\begin{Bmatrix} 1 & 1 \\ \varepsilon & \varepsilon^* \end{Bmatrix}$	$\begin{Bmatrix} 1 & 1 \\ \varepsilon & \varepsilon^* \end{Bmatrix}$	$\begin{Bmatrix} 1 & 1 \\ \varepsilon & \varepsilon^* \end{Bmatrix}$	$\begin{Bmatrix} 1 & 1 \\ \varepsilon & \varepsilon^* \end{Bmatrix}$	$\begin{Bmatrix} 1 & 1 \\ \varepsilon & \varepsilon^* \end{Bmatrix}$	$\begin{Bmatrix} x^2 + y^2, z^2 \\ (x, y), (R_x, R_y) \end{Bmatrix}$
E_2	$\begin{Bmatrix} 1 & i & -1 & -i \\ 1 & -i & -1 & i \end{Bmatrix}$	$\begin{Bmatrix} \varepsilon^* & 1 \\ \varepsilon & \varepsilon^* \end{Bmatrix}$	$\begin{Bmatrix} 1 & 1 \\ \varepsilon & \varepsilon^* \end{Bmatrix}$	$\begin{Bmatrix} 1 & 1 \\ \varepsilon & \varepsilon^* \end{Bmatrix}$	$\begin{Bmatrix} 1 & 1 \\ \varepsilon & \varepsilon^* \end{Bmatrix}$	$\begin{Bmatrix} 1 & 1 \\ \varepsilon & \varepsilon^* \end{Bmatrix}$	$\begin{Bmatrix} 1 & 1 \\ \varepsilon & \varepsilon^* \end{Bmatrix}$	$\begin{Bmatrix} 1 & 1 \\ \varepsilon & \varepsilon^* \end{Bmatrix}$	$(x^2 - y^2, xy)$
E_3	$\begin{Bmatrix} 1 & -\varepsilon^* & -i & \varepsilon \\ 1 & -\varepsilon & i & \varepsilon^* \end{Bmatrix}$	$\begin{Bmatrix} \varepsilon^* & 1 \\ \varepsilon & \varepsilon^* \end{Bmatrix}$	$\begin{Bmatrix} 1 & 1 \\ \varepsilon & \varepsilon^* \end{Bmatrix}$	$\begin{Bmatrix} 1 & 1 \\ \varepsilon & \varepsilon^* \end{Bmatrix}$	$\begin{Bmatrix} 1 & 1 \\ \varepsilon & \varepsilon^* \end{Bmatrix}$	$\begin{Bmatrix} 1 & 1 \\ \varepsilon & \varepsilon^* \end{Bmatrix}$	$\begin{Bmatrix} 1 & 1 \\ \varepsilon & \varepsilon^* \end{Bmatrix}$	$\begin{Bmatrix} 1 & 1 \\ \varepsilon & \varepsilon^* \end{Bmatrix}$	(xz, yz)

► The C_{nv} Groups

C_{2v}	E	C_2	$\sigma_v(xz)$	$\sigma_v'(yz)$	
A_1	1	1	1	1	z
A_2	1	1	-1	-1	R_z
B_1	1	-1	1	-1	x, R_y
B_2	1	-1	-1	1	y, R_x

C_{3v}	E	$2C_3$	$3\sigma_v$	
A_1	1	1	1	z
A_2	1	1	-1	R_z
E	2	-1	0	$(x, y), (R_x, R_y)$

C_{4v}	E	$2C_4$	C_2	$2\sigma_v$	$2\sigma_d$		
A_1	1	1	1	1	1	z	$x^2 + y^2, z^2$
A_2	1	1	1	-1	-1	R_z	
B_1	1	-1	1	1	-1		$x^2 - y^2$
B_2	1	-1	1	-1	1		xy
E	2	0	-2	0	0	$(x, y), (R_x, R_y)$	(xz, yz)

C_{3v}	E	$2C_3$	$2C_2$	$3\sigma_v$		
A_1	1	1	1	1	z	$x^2 + y^2, z^2$
A_2	1	1	1	-1	R_z	
E_1	2	$2 \cos 72^\circ$	$2 \cos 144^\circ$	0	$(x, y), (R_x, R_y)$	(xz, yz)
E_2	2	$2 \cos 144^\circ$	$2 \cos 72^\circ$	0		$(x^2 - y^2, xy)$

C_{6v}	E	$2C_6$	$2C_3$	C_2	$3\sigma_v$	$3\sigma_d$		
A_1	1	1	1	1	1	1	z	$x^2 + y^2, z^2$
A_2	1	1	1	1	-1	-1	R_z	
B_1	1	-1	1	-1	1	-1		
B_2	1	-1	1	-1	-1	1		
E_1	2	1	-1	-2	0	0	$(x, y), (R_x, R_y)$	(xz, yz)
E_2	2	-1	-1	2	0	0		$(x^2 - y^2, xy)$

► The C_{nh} Groups

C_{2h}	E	C_2	i	σ_h		
A_g	1	1	1	1	R_z	x^2, y^2, z^2, xy
B_g	1	-1	1	-1	R_x, R_y	xz, yz
A_u	1	1	-1	-1	z	
B_u	1	-1	-1	1	x, y	

C_{3h}	E	C_3	C_3^2	σ_h	S_3	S_3^2	$\varepsilon = \exp(2\pi i/3)$	
A'	1	1	1	1	1	1	R_z	$x^2 + y^2, z^2$
E'	$\begin{Bmatrix} 1 & \varepsilon & \varepsilon^* \\ 1 & \varepsilon^* & \varepsilon \end{Bmatrix}$	$\begin{Bmatrix} \varepsilon & 1 \\ \varepsilon^* & \varepsilon \end{Bmatrix}$	$\begin{Bmatrix} \varepsilon^* & \varepsilon \\ \varepsilon & \varepsilon^* \end{Bmatrix}$	$\begin{Bmatrix} 1 & \varepsilon & \varepsilon^* \\ 1 & \varepsilon^* & \varepsilon \end{Bmatrix}$	$\begin{Bmatrix} \varepsilon & 1 \\ \varepsilon^* & \varepsilon \end{Bmatrix}$	$\begin{Bmatrix} \varepsilon^* & \varepsilon \\ \varepsilon & \varepsilon^* \end{Bmatrix}$	(x, y)	$(x^2 - y^2, xy)$
A''	1	1	1	-1	-1	-1	z	
E''	$\begin{Bmatrix} 1 & \varepsilon & \varepsilon^* \\ 1 & \varepsilon^* & \varepsilon \end{Bmatrix}$	$\begin{Bmatrix} \varepsilon & 1 \\ \varepsilon^* & \varepsilon \end{Bmatrix}$	$\begin{Bmatrix} \varepsilon^* & \varepsilon \\ \varepsilon & \varepsilon^* \end{Bmatrix}$	$\begin{Bmatrix} 1 & \varepsilon & \varepsilon^* \\ 1 & \varepsilon^* & \varepsilon \end{Bmatrix}$	$\begin{Bmatrix} \varepsilon & 1 \\ \varepsilon^* & \varepsilon \end{Bmatrix}$	$\begin{Bmatrix} \varepsilon^* & \varepsilon \\ \varepsilon & \varepsilon^* \end{Bmatrix}$	(R_x, R_y)	(xz, yz)

Character Tables

D_{4d}	E	$2S_8$	$2C_4$	$2S_4^3$	C_2	$4C_2'$	$4\sigma_d$	(x axis coincident with C_2')	
A_1	1	1	1	1	1	1	1	R_z	$x^2 + y^2, z^2$
A_2	1	1	1	1	1	-1	-1		
B_1	1	-1	1	-1	1	1	-1		
B_2	1	-1	1	-1	1	-1	1	z	
E_1	2	$\sqrt{2}$	0	$-\sqrt{2}$	-2	0	0	(x, y)	
E_2	2	0	-2	0	2	0	0		$(x^2 - y^2, xy)$
E_3	2	$-\sqrt{2}$	0	$\sqrt{2}$	-2	0	0	(R_x, R_y)	(xz, yz)

D_{6d}	1	$2C_3$	$2C_2$	$5C_2$	i	$2S_6^5$	$2S_6$	$5\sigma_d$	(x axis coincident with C_2)	
A_{1g}	1	1	1	1	1	1	1	1		$x^2 + y^2, z^2$
A_{2g}	1	1	1	-1	1	1	1	-1	R_z	
E_{1g}	2	$2 \cos 72^\circ$	$2 \cos 144^\circ$	0	2	$2 \cos 72^\circ$	$2 \cos 144^\circ$	0	(R_x, R_y)	(xz, yz)
E_{2g}	2	$2 \cos 144^\circ$	$2 \cos 72^\circ$	0	2	$2 \cos 144^\circ$	$2 \cos 72^\circ$	0		$(x^2 - y^2, xy)$
A_{1u}	1	1	1	1	-1	-1	-1	-1		
A_{2u}	1	1	1	-1	-1	-1	-1	1	z	
E_{1u}	2	$2 \cos 72^\circ$	$2 \cos 144^\circ$	0	-2	$-2 \cos 72^\circ$	$-2 \cos 144^\circ$	0	(x, y)	
E_{2u}	2	$2 \cos 144^\circ$	$2 \cos 72^\circ$	0	-2	$-2 \cos 144^\circ$	$-2 \cos 72^\circ$	0		

D_{6d}	E	$2S_{12}$	$2C_6$	$2S_4$	$2C_3$	$2S_6^5$	C_2	$6C_2'$	$6\sigma_d$	(x axis coincident with C_2)	
A_1	1	1	1	1	1	1	1	1	1	R_z	$x^2 + z^2, z^2$
A_2	1	1	1	1	1	1	1	-1	-1		
B_1	1	-1	1	-1	1	-1	1	1	-1		
B_2	1	-1	1	-1	1	-1	1	-1	1	z	
E_1	2	$\sqrt{3}$	1	0	-1	$-\sqrt{3}$	-2	0	0	(x, y)	
E_2	2	1	-1	-2	-1	1	2	0	0		$(x^2 - y^2, xy)$
E_3	2	0	-2	0	2	0	-2	0	0		
E_4	2	-1	-1	2	-1	-1	2	0	0		
E_5	2	$-\sqrt{3}$	1	0	-1	$\sqrt{3}$	-2	0	0	(R_x, R_y)	(xz, yz)

THE CUBIC GROUPS

► Tetrahedral Groups

T	E	$4C_3$	$4C_3^2$	$3C_2$	$\varepsilon = \exp(2\pi i/3)$	
A	1	1	1	1		$x^2 + y^2 + z^2$
E	$\begin{Bmatrix} 1 & \varepsilon & \varepsilon^* \\ 1 & \varepsilon^* & \varepsilon \end{Bmatrix}$	$\begin{Bmatrix} \varepsilon & 1 \\ \varepsilon^* & \varepsilon \end{Bmatrix}$	$\begin{Bmatrix} \varepsilon^* & \varepsilon \\ \varepsilon & \varepsilon^* \end{Bmatrix}$	$\begin{Bmatrix} 1 & \varepsilon & \varepsilon^* \\ 1 & \varepsilon^* & \varepsilon \end{Bmatrix}$		$(2z^2 - x^2 - y^2, x^2 - y^2)$
T	3	0	0	-1	$(R, R_y, R_z), (x, y, z)$	(xy, xz, yz)

THE DIHEDRAL GROUPS

► The D_n Groups

D_2	E	$C_2(z)$	$C_2(y)$	$C_2(x)$		
A	1	1	1	1		x^2, y^2, z^2
B_1	1	1	-1	-1	z, R_z	xy
B_2	1	-1	1	-1	y, R_y	xz
B_3	1	-1	-1	1	x, R_x	yz

D_3	E	$2C_3$	$3C_2$	(x axis is coincident with C_2)		
A_1	1	1	1			$x^2 + y^2, z^2$
A_2	1	1	-1	z, R_z		
E	2	-1	0	$(x, y), (R_x, R_y)$		$(x^2 - y^2, xy), (xz, yz)$

D_4	E	$2C_4$	$C_2(=C_4^2)$	$2C_2'$	$2C_2''$	(x axis coincident with C_2')	
A_1	1	1	1	1	1		$x^2 + y^2, z^2$
A_2	1	1	1	-1	-1	z, R_z	
B_1	1	-1	1	1	-1		$x^2 - y^2$
B_2	1	-1	1	-1	1		xy
E	2	0	-2	0	0	$(x, y), (R_x, R_y)$	(xz, yz)

D_5	E	$2C_5$	$2C_3$	$5C_2$	(x axis coincident with C_2)		
A_1	1	1	1	1			$x^2 + y^2, z^2$
A_2	1	1	1	-1	z, R_z		
E_1	2	$2 \cos 72^\circ$	$2 \cos 144^\circ$	0	$(x, y), (R_x, R_y)$		(xz, yz)
E_2	2	$2 \cos 144^\circ$	$2 \cos 72^\circ$	0			$(x^2 - y^2, xy)$

D_6	E	$2C_6$	$2C_3$	C_2	$3C_2'$	$3C_2''$	(x axis coincident with C_2')	
A_1	1	1	1	1	1	1		$x^2 + y^2, z^2$
A_2	1	1	1	1	-1	-1	z, R_z	
B_1	1	-1	1	-1	1	-1		
B_2	1	-1	1	-1	-1	1		
E_1	2	1	-1	-2	0	0	$(x, y), (R_x, R_y)$	(xz, yz)
E_2	2	-1	-1	2	0	0		$(x^2 - y^2, xy)$

► The D_{nh} Groups

D_{2h}	E	$C_2(z)$	$C_2(y)$	$C_2(x)$	i	$\sigma(xy)$	$\sigma(xz)$	$\sigma(yz)$		
A_g	1	1	1	1	1	1	1	1		x^2, y^2, z^2
B_{1g}	1	1	-1	-1	1	1	-1	-1	R_z	xy
B_{2g}	1	-1	1	-1	1	-1	1	-1	R_y	xz
B_{3g}	1	-1	-1	1	1	-1	-1	1	R_x	yz
A_u	1	1	1	1	-1	-1	-1	-1		
B_{1u}	1	1	-1	-1	-1	-1	1	1	z	
B_{2u}	1	-1	1	-1	-1	1	-1	1	y	
B_{3u}	1	-1	-1	1	-1	1	1	-1	x	

D_{3h}	E	$2C_3$	$3C_2$	σ_h	$2S_3$	$3\sigma_v$	(x axis coincident with C_2)		
A_1'	1	1	1	1	1	1			$x^2 + y^2, z^2$
A_2'	1	1	-1	1	1	-1	R_z		
E'	2	-1	0	2	-1	0	(x, y)		$(x^2 - y^2, xy)$
A_1''	1	1	1	-1	-1	-1			
A_2''	1	1	-1	-1	-1	1	z		
E''	2	-1	0	-2	1	0	(R_x, R_y)		(xz, yz)

D_{4h}	E	$2C_4$	C_2	$2C_2'$	$2C_2''$	i	$2S_4$	σ_h	$2\sigma_v$	$2\sigma_d$	(x axis coincident with C_2')	
A_{1g}	1	1	1	1	1	1	1	1	1	1		$x^2 + y^2, z^2$
A_{2g}	1	1	1	-1	-1	1	1	1	-1	-1	R_z	
B_{1g}	1	-1	1	1	-1	1	-1	1	1	-1		$x^2 - y^2$
B_{2g}	1	-1	1	-1	1	1	-1	1	-1	1		xy
E_g	2	0	-2	0	0	2	0	-2	0	0	(R_x, R_y)	(xz, yz)
A_{1u}	1	1	1	1	1	-1	-1	-1	-1	-1		
A_{2u}	1	1	1	-1	-1	-1	-1	-1	1	1	z	
B_{1u}	1	-1	1	1	-1	-1	1	-1	1	-1		
B_{2u}	1	-1	1	-1	1	-1	1	-1	-1	1		
E_u	2	0	-2	0	0	-2	0	2	0	0	(x, y)	

D_{5h}	E	$2C_5$	$2C_3$	$5C_2$	σ_h	$2S_5$	$2S_3$	$5\sigma_v$	(x axis coincident with C_2)	
A_1'	1	1	1	1	1	1	1	1		$x^2 + y^2, z^2$
A_2'	1	1	1	-1	1	1	1	-1	R_z	
E_1'	2	$2 \cos 72^\circ$	$2 \cos 144^\circ$	0	2	$2 \cos 72^\circ$	$2 \cos 144^\circ$	0	(x, y)	
E_2'	2	$2 \cos 144^\circ$	$2 \cos 72^\circ$	0	2	$2 \cos 144^\circ$	$2 \cos 72^\circ$	0		$(x^2 - y^2, xy)$
A_1''	1	1	1	1	-1	-1	-1	-1		
A_2''	1	1	1	-1	-1	-1	-1	1	z	
E_1''	2	$2 \cos 72^\circ$	$2 \cos 144^\circ$	0	-2	$-2 \cos 72^\circ$	$-2 \cos 144^\circ$	0	(R_x, R_y)	(xz, yz)
E_2''	2	$2 \cos 144^\circ$	$2 \cos 72^\circ$	0	-2	$-2 \cos 144^\circ$	$-2 \cos 72^\circ$	0		